BC Calculus Summer Work

To keep alive the techniques and knowledge you amassed last school year in AB Calculus, I am asking that you take two AB tests this summer. The first one is presented here and will be posted around July 1st. The second one will be posted around August 1st.

These AB tests are to completed and ready to discuss on the first day of class. **Write down your work for each problem**. They will be graded for completeness, not correctness. Although you do not need to take them under AP conditions, you may want to do so.

Here's my suggestion: Try taking them under AP conditions and then go back and do the ones you could not do as an open book test.

If you get stuck and want to discuss something, please e-mail me at sarah.lucic@woosterschool.org.

Good luck!

AP Calculus Practice Exam
AB Version - Section I - Part A

Calculators ARE NOT Permitted On This Portion Of The Exam
28 Questions - 55 Minutes

1) Give $f(g(1))$, given that

$$\left[f(x) = 2x + 2,\ g(x) = -\frac{x}{2 + x^2}\right]$$

a) $\frac{-8}{9}$

b) $\frac{7}{3}$

c) 2

d) $\frac{4}{3}$

e) $\frac{-2}{9}$

2) Find the slope of the tangent line to the graph of f at $x = 4$, given that

$$f(x) = -x^2 + 4\sqrt{x}$$

a) -8

b) -10

c) -9

d) -5

e) -7

3) Determine

$$\lim_{x \to \infty} \left(\frac{-2x^3 + x}{-4x^5 + 2x^2 + 2} \right)$$

a) ∞

b) 0

c) $\frac{1}{2}$

d) $\frac{3}{10}$

e) 1

4) Let

$$f(x) = x^3$$

A region is bounded between the graphs of $y = -1$ and $y = f(x)$ for x between -1 and 0, and between the graphs of $y = 1$ and $y = f(x)$ for x between 0 and 1. Give an integral that corresponds to the area of this region.

a) $\int_{-1}^{1} \left(1 - x^3\right) dx$

b) $\int_{0}^{1} 2\left(1 - x^3\right) dx$

c) $\int_{0}^{1} 2\left(1 + x^3\right) dx$

d) $\int_{-1}^{1} \left(1 + x^3\right) dx$

e) $\int_{0}^{1} \left(-x^3 - 1\right) dx$

5) Given that

$$5x^3 - 4xy - 2y^2 = 1$$

Determine the change in y with respect to x.

a) $-\dfrac{15x^2 - 4}{-4 - 4y}$

b) $-\dfrac{15x^2 - 4y}{-4 - 4y}$

c) $-\dfrac{15x^2 - 4}{-4x - 4y}$

d) $-\dfrac{10x - 4y}{-4x - 2}$

e) $-\dfrac{15x^2 - 4y}{-4x - 4y}$

6) Compute the derivative of

$$-4\sec(x) + 2\csc(x)$$

a) $-4\sec(x)\tan(x) - 2\csc(x)\cot(x)$

b) $-4\csc(x) - 2\sec(x)$

c) $-4(\sec(x))^2 - 2(\csc(x))^2$

d) $-4\sec(x)\tan(x) + 2\csc(x)\cot(x)$

e) $-4(\tan(x))^2 - 2(\cot(x))^2$

7) Compute

$$\int_0^{\frac{1}{2}} \frac{4}{1+4t^2}\,dt$$

a) $-\pi$

b) $\frac{3}{2}\pi$

c) $\frac{1}{2}\pi$

d) π

e) 0

8) Determine

$$\frac{d}{dx}\left(\frac{4x^4-2x}{4x^4+2x}\right)$$

a) $\frac{24x^2-1}{\left(4x^3+2\right)^2}$

b) $\frac{48x^2-1}{\left(4x^3+2\right)^2}$

c) $\frac{12x^2}{\left(2x^3+1\right)^2}$

d) $\frac{24x^2}{\left(4x^3+2\right)^2}$

e) $\frac{6x^2}{\left(4x^3+2\right)^2}$

9) Give the equation of the normal line to the graph of

$$y = 2x\sqrt{x^2+8}+2$$

at the point (0 , 2).

a) $x - 4\sqrt{2}\,y = -8\sqrt{2}$

b) $x + 4\sqrt{2}\,y = 8\sqrt{2}$

c) $4\sqrt{2}\,x + y = 2$

d) $-4\sqrt{2}\,x + y = 2$

e) $x + 4\sqrt{2}\,y = 2$

10) Determine the concavity of the graph of

$$f(x) = 3\sin(x) + 4(\cos(x))^2$$

at $x = \pi$.

a) 8

b) -10

c) 4

d) -8

e) -6

11) Compute

$$\int 4x^2\sqrt{x^3+4}\,dx$$

a) $\frac{8}{3}\left(x^3+4\right)^{(3/2)} + C$

b) $\frac{16}{9}\left(x^3+4\right)^{(3/2)} + C$

c) $\frac{8}{9}\left(x^3+4\right)^{(3/2)} + C$

d) $\frac{4}{3}\frac{1}{\sqrt{x^3+4}} + C$

e) $\frac{8}{3}\frac{1}{\sqrt{x^3+4}} + C$

12) Give the value of x where the function

$$f(x) = x^3 - 9x^2 + 24x + 4$$

has a local maximum.

a) 4

b) -2

c) 2

d) -4

e) 3

13) The slope of the tangent line to the graph of

$$4x^2 + cx - 2e^y = -2$$

at $x = 0$ is 4. Give the value of c.

a) -2

b) 4

c) 8

d) -4

e) -8

14) Compute

$$\int \left(5^{x} + 2\, e^{(5 \ln(x))}\right) dx$$

a) $\dfrac{5^{x}}{\ln(5)} + \dfrac{2}{5} e^{(5 \ln(x))} + C$

b) $5^{x} \ln(5) + \dfrac{2}{5} e^{(5 \ln(x))} + C$

c) $5^{x} \ln(5) + \dfrac{2}{5} \dfrac{e^{(5 \ln(x))}}{x} + C$

d) $\dfrac{5^{x}}{\ln(5)} + \dfrac{2}{5} x^{5} + C$

e) $\dfrac{5^{x}}{\ln(5)} + \dfrac{1}{3} x^{6} + C$

15) What is the average value of the function

$$g(x) = (2x + 3)^{2}$$

on the interval from $x = -3$ to $x = -1$?

a) $\dfrac{7}{3}$

b) -4

c) 5

d) $\dfrac{14}{3}$

e) 3

16) Compute

$$\lim_{t \to 0} \left(\frac{\tan\left(\frac{1}{4}\pi + t\right) - \tan\left(\frac{1}{4}\pi\right)}{t} \right)$$

a) 1

b) $\frac{1}{4}\pi$

c) π

d) 2

e) -1

17) Find the instantaneous rate of change of

$$f(t) = \left(2t^3 - 3t + 4\right)\sqrt{t^2 + 3t + 4}$$

at $t = 0$.

a) -3

b) $\frac{-3}{4}$

c) 0

d) -4

e) $\frac{-5}{4}$

18) Compute

$$\frac{d}{dx} 2^{\cos(x)}$$

a) $\sin(x)\ 2^{\cos(x)} \ln(2)$

b) $-\sin(x)\ 2^{\cos(x)} \ln(2)$

c) $-\sin(x)\ 2^{\cos(x)}$

d) $-\dfrac{\sin(x)\ 2^{\cos(x)}}{\ln(2)}$

e) $\dfrac{\sin(x)\ 2^{\cos(x)}}{\ln(2)}$

19) A solid is generated by rotating the region enclosed by the graph of

$$y = \sqrt{x}$$

the lines $x = 1$, $x = 2$, and $y = 1$, about the x-axis. Which of the following integrals gives the volume of the solid?

a) $\int_1^2 \pi\ (x - 1)\ dx$

b) $\int_1^2 \pi\ (x - 1)^2\ dx$

c) $\int_1^2 \pi\ \left(\sqrt{x} - 1\right)^2\ dx$

d) $\int_1^2 \pi\ (2 - x)^2\ dx$

e) $\int_1^2 \pi\ \left(2 - \sqrt{x}\right)^2\ dx$

20) Compute

$$\lim_{x \to 0} \left(-\frac{4x}{\sin(2x)} + \frac{x}{\cos(2x)} \right)$$

a) ∞

b) 0

c) $\frac{-5}{2}$

d) -2

e) *undefined*

21) Given $y > 0$ and

$$\frac{dy}{dx} = \frac{3x^2 + 4x}{y}$$

If the point

$$\left(1, \sqrt{10}\right)$$

is on the graph relating x and y, then what is y when $x = 0$?

a) 3

b) 2

c) 1

d) 6

e) 10

22) Determine

$$\int_1^2 \frac{1}{\sqrt{4-t^2}}\,dt$$

a) $\frac{1}{2}\pi$

b) $\frac{1}{3}\pi$

c) π

d) $\frac{1}{6}\pi$

e) $\frac{1}{4}\pi$

23) Determine

$$\int e^{(2x)}\sqrt{e^x+1}\,dx$$

a) $\frac{2}{5}\left(e^x+1\right)^{(5/2)} - \frac{2}{3}\left(e^x+1\right)^{(3/2)} + C$

b) $e^{(2x)}\left(e^x+1\right)^{(3/2)} + C$

c) $\frac{2}{5}e^{\left(\frac{5}{2}x\right)} - 5e^{\left(\frac{3}{2}x\right)} + C$

d) $\frac{2}{5}\left(e^x+1\right)^{(5/2)} - 3\left(e^x+1\right)^{(3/2)} + C$

e) $\frac{2}{5}\left(e^x+1\right)^{(5/2)} + 3\left(e^x+1\right)^{(3/2)} + C$

24) A particle's acceleration for $t \geq 0$ is given by

$$a(t) = 12t + 4$$

The particle's initial position is 2 and its velocity at $t = 1$ is 5. What is the position of the particle at $t = 2$?

a) 10

b) 12

c) 16

d) 4

e) 20

25) Determine

$$\int_0^{\frac{1}{2}\pi} \sin(3x)\,dx + \int_0^{\frac{1}{6}\pi} \cos(3x)\,dx$$

a) -1

b) 1

c) 0

d) $\frac{2}{3}$

e) $\frac{-2}{3}$

26) Determine the derivative of

$$f(x) = (\cos(2x - 4))^3$$

at $x = \pi/2$.

a) $-6\ (\cos(\pi - 4))^2$

b) $-6\cos(\pi - 4)^2 \sin(\pi - 4)$

c) $-6\ (\cos(\pi - 4))^2 \sin(\pi - 4)$

d) $18\ (\cos(\pi - 4))^2 \sin(\pi - 4)$

e) $18\ (\cos(\pi - 4))^2$

27) Compute the derivative of

$$f(x) = \int_0^{x^2} \ln(t^2 + 1)\ dt$$

a) $\ln(x^4 + 1)$

b) $2x\ln(x^4 + 1)$

c) $\dfrac{2x}{x^4 + 1}$

d) $2x\ln(x^2 + 1)$

e) $\ln(x^2 + 1)$

28) Determine

$$\frac{d}{dx}\ln(\ln(2-\cos(x)))$$

a) $\dfrac{\cos(x)}{(2-\cos(x))\ln(2-\cos(x))}$

b) $\dfrac{\sin(x)}{\ln(2-\cos(x))}$

c) $\dfrac{\sin(x)}{(2-\cos(x))\ln(2-\cos(x))}$

d) $\dfrac{\sin(x)\ (2-\cos(x))}{\ln(2-\cos(x))}$

e) $-\dfrac{\cos(x)}{\ln(2-\cos(x))}$

AP Calculus Practice Exam
AB Version - Section I - Part B

Calculators ARE Permitted On This Portion Of The Exam
17 Questions - 50 Minutes

1) Give a value of c that satisfies the conclusion of the Mean Value Theorem for Derivatives for the function

$$f(x) = -2x^2 - x + 2$$

on the interval [1,3].

a) $\frac{9}{4}$

b) $\frac{3}{2}$

c) $\frac{1}{2}$

d) 2

e) $\frac{5}{4}$

2) The function

$$f(x) = 3x^3 + 2e^{(2x)}$$

is invertible. Give the derivative of f^{-1} at $x = 2$.

a) $9 + 4e^2$

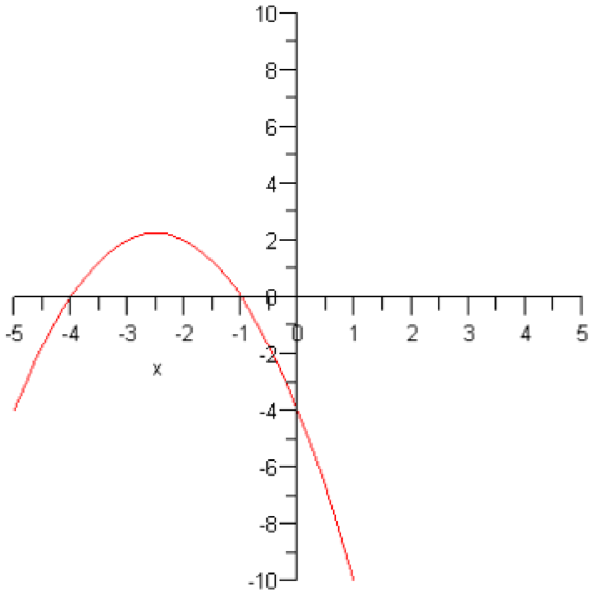
b) 4

c) $\frac{1}{9 + 4e^2}$

d) 1

e) $\frac{1}{4}$

3) The **derivative** of f is graphed below.



Give a value of x where f has a local maximum.

a) -4

b) -1

c) $\frac{-5}{2}$

d) *There is no such value of* x.

e) 1

4) Let

$$f(x) = \begin{cases} -x+5 & x < -2 \\ x^2+1 & -2 \le x \text{ and } x \le 1 \\ 2x^3-1 & 1 \le x \end{cases}$$

Which of the following is (are) true?

1) f is continuous at x = -2.
2) f is differentiable at x = 1.
3) f has a local minimum at x = 0.
4) f has an absolute maximum at x = -2.

a) 2 and 4
b) 3 only
c) 2 only
d) 1 and 3
e) 1 and 4

5) Given

$$\left[\int_0^{50} 3\, f(x)\, dx = 3,\ \int_2^{50} f(x)\, dx = -4\right]$$

Determine

$$\int_0^2 f(x)\, dx$$

a) 10

b) -3

c) *There is not enough information.*

d) -6

e) 5

6) Give the approximate location of a local maximum for the function

$$f(x) = 3x^3 + 5x^2 - 3x$$

a) $(-1.357, 5.779)$

b) $(0.2457, -.3908)$

c) $(-1.357, 5.713)$

d) $(0.2457, -.3216)$

e) $(-1.357, -.3908)$

7) Give the approximate average value of the function

$$f(x) = 4x\ln(2x)$$

over the interval [1,4].

a) 19.71

b) 12.54

c) 16.71

d) 18.02182670

e) 18.71

8) The region enclosed by the graphs of

$$[y = x^3 - 1,\ y = x - 1]$$

is rotated around the y-axis to generate a solid. What is the volume of the solid?

a) 0.8380

b) 0.7855

c) 1.676

d) 1.047

e) 2.356

9) What is the approximate instantaneous rate of change of the function

$$f(t) = \int_0^{8t} \cos(x)\, dx$$

at $t = \pi/7$?

a) $-.9009$

b) -7.207

c) 3.473

d) 0.4341

e) -1.030

10) What is the error when the integral

$$\int_0^1 \sin(\pi x)\, dx$$

is approximated by the Trapezoidal rule with $n = 3$?

a) 0.011

b) 0.032

c) 0.109

d) 0.059

e) 0.051

11) The amount of money in a bank account is increasing at the rate of

$$R(t) = 10000\, e^{(0.06\, t)}$$

dollars per year, where t is measured in years. If $t = 0$ corresponds to the year 2005, then what is the approximate total amount of increase from 2005 to 2007.

a) \$18,350
b) \$4,500
c) \$21,250
d) \$32,560
e) \$16,250

12) A particle moves with acceleration

$$a(t) = 3t^2 - 2t$$

and its initial velocity is 0. For how many values of t does the particle change direction?

a) 3

b) 2

c) 1

d) 0

e) 4

13) At what approximate rate (in cubic meters per minute) is the volume of a sphere changing at the instant when the surface area is 5 square meters and the radius is increasing at the rate of 1/3 meters per minute?

a) 5.271

b) 1.700

c) 1.667

d) 1.080

e) 2.714

14) A rectangle has one side on the x-axis and the upper two vertices on the graph of

$$y = e^{(-2x^2)}$$

Give a decimal approximation to the maximum possible area for this rectangle.

a) 1.649

b) 1.

c) $-1.$

d) 0.5458

e) 0.6065

15) A rough approximation for ln(5) is 1.609. Use this approximation and differentials to approximate ln(128/25).

a) 1.633

b) 1.621

c) 1.632

d) 1.585

e) 1.597

16) The function

$$f(x) = \begin{cases} n x^3 - x & x \leq 1 \\ m x^2 + 5 & 1 < x \end{cases}$$

is differentiable everywhere. What is n?

a) -9

b) 13

c) -17

d) -11

e) -14

17) Which of the following functions has a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 2$?

a) $f(x) = \dfrac{2x^2 + 1}{x^2 - 1}$

b) $f(x) = \ln(2x + 2)$

c) $f(x) = e^{(x-1)} + 2$

d) $f(x) = \arctan(x - 1) + 2 - \dfrac{1}{2}\pi$

e) $f(x) = \dfrac{x - 1}{2x + 2}$

SECTION II

Time: 1 hour and 30 minutes
Percent of total grade: 50

Part A: 45 minutes, 3 problems
(A graphing calculator is required for some problems or parts of problems.)

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Part B: 45 minutes, 3 problems
(No calculator is allowed for these problems.)

During the timed portion for Part B, you may keep Part A, and continue to work on the problems in Part A without the use of any calculator.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND PART B
For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM WITH A PENCIL OR PEN IN THE SPACE PROVIDED FOR THAT PART IN THE PINK EXAM BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB
SECTION II, Part A
Time --- 45 minutes
Number of Problems --- 3

A graphing calculator is required for some problems or parts of the problems.

1. Let *S* be the region in the first quadrant bounded by the graphs of $y = e^{-x^2}$ and $y = 2x^2$, and the *y*-axis.

a) Find the area of the region *S*.

b) Find the volume of the solid generated when the region *S* is rotated about the x-axis.

c) The region *S* is the base of a solid for which each cross section perpendicular to the *x*-axis is a semi-circle with diameter in the xy plane. Find the volume of this solid.

2. Let $F(x)=\int_0^x \sin^2(t)\,dt$ on the interval $[0,\pi]$.

a) Approximate $F(\pi)$ using the trapezoid rule with n = 4.

b) Find $F'(x)$.

c) Find the average value of $F'(x)$ on the interval $[0,\pi]$.

3. A particle moves along the *x*-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t - 4$. At time $t = 1$, the velocity of the particle is $v(1) = 7$ and its position is $x(1) = 4$.

a) Write an expression for the velocity of the particle $v(t)$.
b) At what values of *t* does the particle change direction?
c) Write an expression for the position $x(t)$ of the particle.
d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

CALCULUS AB
SECTION II, Part B
Time --- 45 minutes
Number of Problems --- 3

No Calculator is allowed for these problems.

4. Consider the graph of $f(x) = x^4 - 6x^2$.

a) Find the relative maxima and minima (both *x* and *y* coordinates).
b) Find the coordinates of the point(s) of inflection.
c) Determine the interval(s) on which the function is increasing.
d) Determine the interval(s) on which the function is concave up.

5. Consider the equation $x^2 + 3y^2 + xy = 3$.

a) Write an expression for the slope of the curve at any point (x, y).

b) Find the equation of the normal line to the curve at the point $(0,1)$.

c) What is $\frac{d^2y}{dx^2}$ at $(0,1)$?

d) If $(a, -2a)$ is a point on the curve, determine the nature of the line tangent to the curve at that point.

6. Water is poured into a tank in the shape of a right circular cone, standing on its vertex. The height of the cone is 40 feet and its radius is 8 feet. The water level in the tank is increasing at a constant rate of 2 feet per second.

a) Find an expression for the volume of the water (in ft^3) in the tank in terms of its height.

b) How fast (in ft/sec) is the water being poured into the tank at the instant the depth of the water is 10 feet?

c) How fast is the area of the surface of the water increasing at the instant the depth of the water is 10 feet?

END OF EXAM

BC Calculus Summer Work

To keep alive the techniques and knowledge you amassed last school year in AB Calculus, I am asking that you take two AB tests this summer. The first one was posted around July 1st. The second one is presented here and will be posted around August 1st.

These AB tests are to completed and ready to discuss on the first day of class. **Write down your work for each problem**. They will be graded for completeness, not correctness. Although you do not need to take them under AP conditions, you may want to do so.

Here's my suggestion: Try taking them under AP conditions and then go back and do the ones you could not do as an open book test.

If you get stuck and want to discuss something, please give me a call on my cell at (203)947-9574.

Good luck!

Have a great summer,
AEH

1988

MATHEMATICS : CALCULUS AB

CALCULATORS AND REFERENCE MATERIALS MAY NOT BE USED IN THE EXAMINATION ROOM DURING THE TESTING PERIOD.

Three hours are allotted for this examination: 1 hour and 30 minutes for Section I, which consists of multiple-choice questions; and 1 hour and 30 minutes for Section II, which consists of longer problems. In determining your grade, the two sections are given equal weight. Section I is printed in this examination booklet; Section II, in a separate booklet.

SECTION I

Time — 1 hour and 30 minutes

Number of questions — 45

Percent of total grade — 50

This examination contains 45 multiple-choice questions. Therefore, please be careful to fill in only the ovals that are preceded by numbers 1 through 45 on your answer sheet.

General Instructions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE ALL YOUR ANSWERS TO QUESTIONS IN SECTION I ON THE SEPARATE ANSWER SHEET ENCLOSED. No credit will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Example:

What is the arithmetic mean of the numbers 1, 3, and 6 ?

(A) 1 (B) $\frac{7}{3}$ (C) 3

(D) $\frac{10}{3}$ (E) $\frac{7}{2}$

Sample Answer

Ⓐ Ⓑ Ⓒ ● Ⓔ

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected that everyone will be able to answer all the multiple-choice questions.

CALCULUS AB

SECTION I

Time—1 hour and 30 minutes

Number of questions—45

Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratchwork. Then decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in this examination booklet. Do not spend too much time on any one problem.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = x^2e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$
(B) $x(x + 2e^x)$
(C) $xe^x(x + 2)$
(D) $2x + e^x$
(E) $2x + e$

GO ON TO THE NEXT PAGE

2. What is the domain of the function f given by $f(x) = \dfrac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x:\ x \neq 3\}$
(B) $\{x:\ |x| \leqq 2\}$
(C) $\{x:\ |x| \geqq 2\}$
(D) $\{x:\ |x| \geqq 2 \text{ and } x \neq 3\}$
(E) $\{x:\ x \geqq 2 \text{ and } x \neq 3\}$

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3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$
(B) $e - 1$
(C) $2e$
(D) e^2
(E) $\dfrac{e^3}{3}$

GO ON TO THE NEXT PAGE

4. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x >$

5. $\int \sec^2 x \, dx =$

(A) $\tan x + C$

(B) $\csc^2 x + C$

(C) $\cos^2 x + C$

(D) $\dfrac{\sec^3 x}{3} + C$

(E) $2 \sec^2 x \tan x + C$

GO ON TO THE NEXT PAGE

6. If $y = \dfrac{\ln x}{x}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{x^2}$ (C) $\dfrac{\ln x - 1}{x^2}$ (D) $\dfrac{1 - \ln x}{x^2}$ (E) $\dfrac{1 + \ln x}{x^2}$

---

7. $\displaystyle\int \frac{x\,dx}{\sqrt{3x^2 + 5}} =$

(A) $\frac{1}{9}(3x^2 + 5)^{\frac{3}{2}} + C$ (B) $\frac{1}{4}(3x^2 + 5)^{\frac{3}{2}} + C$ (C) $\frac{1}{12}(3x^2 + 5)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}(3x^2 + 5)^{\frac{1}{2}} + C$ (E) $\frac{3}{2}(3x^2 + 5)^{\frac{1}{2}} + C$

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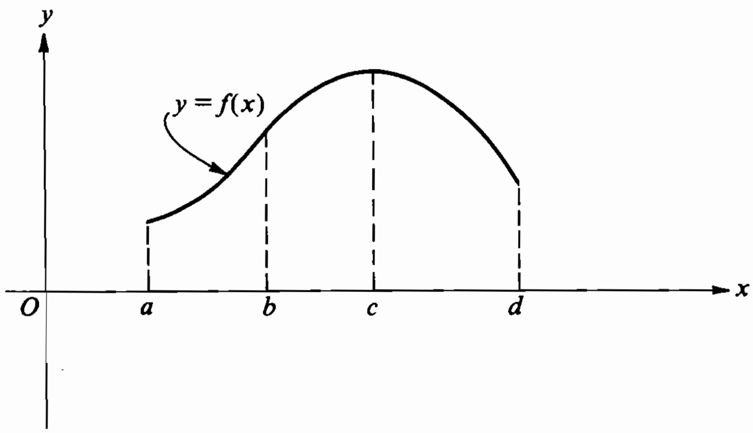

8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

GO ON TO THE NEXT PAGE

9. If $x + 2xy - y^2 = 2$, then at the point $(1, 1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

---

10. If $\int_0^k (2kx - x^2)dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

GO ON TO THE NEXT PAGE

11. An equation of the line tangent to the graph of $f(x) = x(1 - 2x)^3$ at the point $(1, -1)$ is

(A) $y = -7x + 6$

(B) $y = -6x + 5$

(C) $y = -2x$ ·

(D) $y = 2x - 3$

(E) $y = 7x - 8$

---

12. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\frac{\sqrt{3}}{2}$

(E) $\sqrt{3}$

GO ON TO THE NEXT PAGE

13. If the function f has a continuous derivative on $[0, c]$, then $\int_0^c f'(x)\,dx =$

(A) $f(c) - f(0)$ (B) $|f(c) - f(0)|$ (C) $f(c)$ (D) $f(x) + c$ (E) $f''(c) - f''(0)$

---

14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2(\sqrt{2} - 1)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2(\sqrt{2} - 1)$ (E) $2(\sqrt{2} + 1)$

GO ON TO THE NEXT PAGE

15. If $f(x) = \sqrt{2x}$, then $f'(2) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

16. A particle moves along the x-axis so that at any time $t \geqq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

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17. $\int_0^1 (3x - 2)^2 \, dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

---

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

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19. $\int_2^3 \frac{x}{x^2+1}\,dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln$

---

20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

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21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

---

22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

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23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

---

24. $\frac{d}{dx}(x^{\ln x}) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)(x^{\ln x})$ (D) $(\ln x)(x^{\ln x - 1})$ (E) $2(\ln x)(x^{\ln x})$

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25. For all $x > 1$, if $f(x) = \int_1^x \frac{1}{t}\, dt$, then $f'(x) =$

(A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x

---

26. $\int_0^{\frac{\pi}{2}} x \cos x \, dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1 - \frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2} - 1$

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27. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geqq 3 \end{cases}$ is

(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

(E) both continuous and differentiable

---

28. $\int_1^4 |x - 3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

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29. The $\lim_{h\to 0} \dfrac{\tan 3(x+h) - \tan(3x)}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

---

30. A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

(A) $2\pi \int_0^1 xe^{2x}\,dx$

(B) $2\pi \int_0^1 e^{2x}\,dx$

(C) $\pi \int_0^1 e^{4x}\,dx$

(D) $\pi \int_0^e y \ln y\,dy$

(E) $\frac{\pi}{4} \int_0^e \ln^2 y\,dy$

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31. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x

32. Which of the following does NOT have a period of π ?

(A) $f(x) = \sin\left(\frac{1}{2}x\right)$ (B) $f(x) = |\sin x|$ (C) $f(x) = \sin^2 x$
(D) $f(x) = \tan x$ (E) $f(x) = \tan^2 x$

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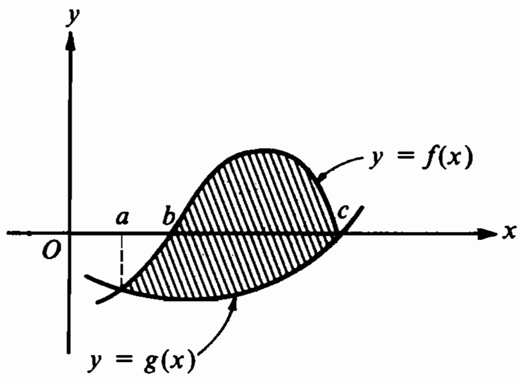
33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2



34. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)| - |g(x)|)dx$

(B) $\int_b^c f(x)dx - \int_a^c g(x)dx$

(C) $\int_a^c (g(x) - f(x))dx$

(D) $\int_a^c (f(x) - g(x))dx$

(E) $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))dx$

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35. $4\cos\left(x + \frac{\pi}{3}\right) =$

(A) $2\sqrt{3}\cos x - 2\sin x$ (B) $2\cos x - 2\sqrt{3}\sin x$ (C) $2\cos x + 2\sqrt{3}$ si

(D) $2\sqrt{3}\cos x + 2\sin x$ (E) $4\cos x + 2$

---

36. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) -6 (B) -2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

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37. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

38. For $x > 0$, $\int \left(\frac{1}{x} \int_1^x \frac{du}{u} \right) dx =$

(A) $\frac{1}{x^3} + C$

(B) $\frac{8}{x^4} - \frac{2}{x^2} + C$

(C) $\ln(\ln x) + C$

(D) $\frac{\ln(x^2)}{2} + C$

(E) $\frac{(\ln x)^2}{2} + C$

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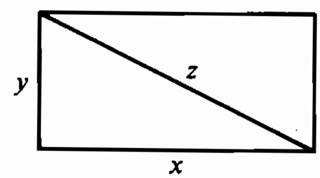
39. If $\int_1^{10} f(x)dx = 4$ and $\int_{10}^{3} f(x)dx = 7$, then $\int_1^{3} f(x)dx =$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11



40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

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41. If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

---

42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$
(B) $y = \sin x$
(C) $y = \frac{x}{x+1}$
(D) $y = \frac{x^2}{x-1}$
(E) $y = e^{-x}$

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43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ abou the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

---

44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II, and III

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MATHEMATICS : CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

It may be worthwhile for you to look through the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. The questions are printed in the booklet and on the green insert. It will be easier for you to look over all the problems on the insert; however, you should write all work for each problem in the space provided for that particular problem in the pink booklet. You may work the problems in any order. Do not spend too much time on any one problem.

Write all your answers in pencil only. Be sure to write CLEARLY and LEGIBLY. If you make an error, you may save time by crossing it out rather than trying to erase it.

Show all your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. All questions are given equal weight, but the parts of a particular question are not necessarily given equal weight. Credit for partial solutions will be given.

When you are told to begin, open your booklet, carefully tear out the green insert, and start work.

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

(a) Find the domain of f.

(b) Describe the symmetry, if any, of the graph of f.

Continue problem 1 on next page.

(c) Find $f'(x)$.

(d) Find the slope of the line normal to the graph of f at $x = 5$.

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2. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

(a) Find the acceleration $a(t)$ of the particle at any time t.

(b) Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.

Continue problem 2 on next page.

(c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

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3. Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

(a) Find the volume of the solid generated by revolving R about the <u>x-axis</u>.

Continue problem 3 on next page.

(b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line $x = -1$.

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4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

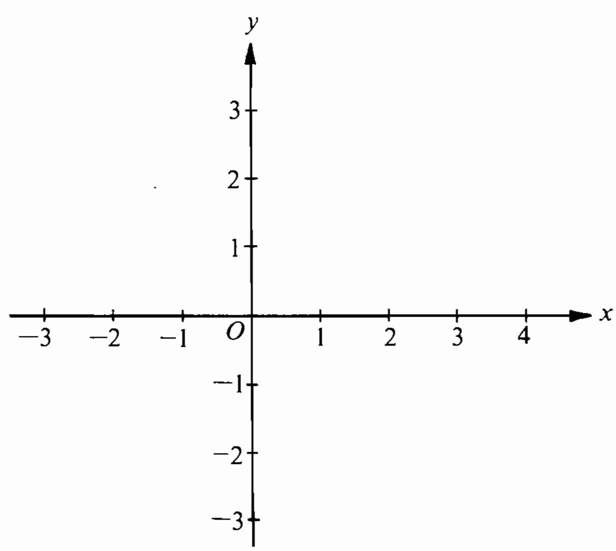
(a) Write an equation of the horizontal asymptote for the graph of f.

(b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.

Continue problem 4 on next page.

(c) For what values of x is the graph of f concave down?

(d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ in the xy-plane provided below.



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5. Let R be the region in the first quadrant under the graph of $y = \frac{x}{x^2 + 2}$ for $0 \leqq x \leqq \sqrt{6}$.

(a) Find the area of R.

(b) If the line $x = k$ divides R into two regions of equal area, what is the value of k?

Continue problem 5 on next page.

(c) What is the average value of $y = \dfrac{x}{x^2 + 2}$ on the interval $0 \leqq x \leqq \sqrt{6}$?

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6. Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x)dx = 18$

Find $f(x)$. Show your work.

If more space is needed, continue problem 6 on next page.