Summer Work
Honors Algebra II

Name: ________________________________

Part I

Honors Algebra Two Students:

The following questions are designed to keep your math skills sharp over the lazy summer months. The summer work packet is divided into four parts, and each part will require about 2 hrs to complete. In addition to providing you with practice while away from school, this review will help me assess your strengths and weaknesses coming into the class. While you are free to use any notes or texts from previous classes, please do not ask anyone else for assistance. Further expectations for the work's completion are outlined below. Take note of those problems with which you have trouble; we will have the opportunity to discuss tricky problems during the first week of class.

Description of assignment: Each of the four parts of the summer work is split into three sections.

1. Review of concepts from algebra – **complete all odd problems**
2. Review of concepts from geometry – **complete all odd problems**
3. Introduction to regression – **complete entirely**

If you really get stuck on one of the assigned problems, do another problem in its place and write a note to me indicating your substitution. You are encouraged to do more than the assigned problems, if your heart desires—the more you practice, the sharper you'll be.

Notes regarding completion and submission:

1. All parts of the summer work must be submitted on the first day of class.
2. You may use a calculator.
3. All work must be done in **pencil**. If you need additional pages to do your work, **you must use graph paper**—sorry, math department policy—and be sure to include these extra pages when submitting your summer work.
4. Please **box** or **circle your answers**.
5. **Show your work!** I am interested in both **accuracy** and **process**.
6. You may use any written resources available to you, but please do not ask another person for assistance.
7. **Grades are based on effort, not correctness**, so give everything your best shot!
8. The grade you earn will play a significant role in your first marking period grade and effort grade.

Algebra Review

1-4 Use an associative property to write an equivalent expression.

1. $(5 \cdot a) \cdot 7$ ______ **2.** $m + (n + 1)$ ______ **3.** $(a + b) + c$ ______

1-5 Use the distributive property to write an equivalent expression.

4. $2(3w + 2x)$ ______ **5.** $3(5x + 3y + 2)$ ______ **6.** $7(a + 3b + c)$ ______

Simplify.

2-4 **7.** $5 - (-8x) - 3 - x$ ______ **8.** $-3 - 2 - (-7)$ ______

9. $1.4 - 1.8 - (-3)$ ______ **10.** $3y - (-2y) - 3y$ ______

2-8 **11.** $7a - (5a - 4)$ ______ **12.** $3a - 2b - 2(2a - b)$ ______

13. $[2(3y + 1) + 5] - 5y$ ______ **14.** $5[9 - 2(3x + 4)]$ ______

3-4 Write as an algebraic expression.

15. The product of two consecutive integers ______

16. The sum of three consecutive even integers ______

17. Seven times the sum of a number and 5 ______

Solve.

3-5 **18.** $6x - 24 = -48$ ______ **19.** $-5t = 81 + 4t$ ______

20. $-10a - 6a = 48$ ______ **21.** $2 - 3x = -4x + 16$ ______

3-8 **22.** $|-3| + |-7| + |c| = 10$ ______ **23.** $|a| - 21 = 15$ ______

24. $2|y| + 10 = 38$ ______ **25.** $3|m| - 5 = 10$ ______

3-9 **26.** Cory works 6 hours to earn \$27. How many hours must he work to earn \$72? ______

4-2 Solve and graph.

27. $x - 3 > 1$ ______

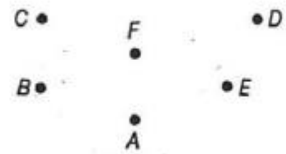
28. $2x + 1 > -5$ ______

29. $3y - 4 - y < -4$ ______

Geometry Review

Directions: Write answers in the spaces provided.

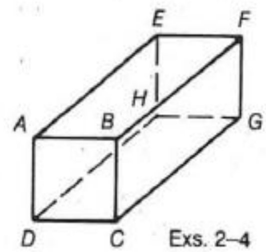
Exercise 1 refers to the diagram at the right.



Ex. 1

1. Name the points that appear(s) to be equidistant from B, C, and E. ____________

Exercises 2–4 refer to the diagram at the right.



Exs. 2–4

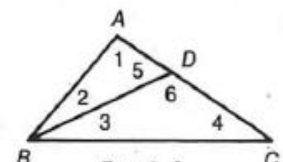
2. Of the lines shown, name the lines that do *not* intersect plane $BCGF$. ____________

3. Identify the intersection of planes $EFGH$ and $AEHD$.

____

4. Are $A, D, G,$ and E coplanar? ____________

Exercises 5 and 6 refer to the diagram at the right.

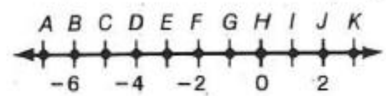


Exs. 5–6

5. $\overrightarrow{BD}$ bisects $\angle ABC$ and $m\angle 2 = 24$. Find $m\angle ABC$. ____________

6. Name the vertex and the sides of $\angle 2$.

vertex ____________ sides ____________



Ex. 7

7. Use the diagram to find the distance from the midpoint of $\overline{AG}$ to K. ____________

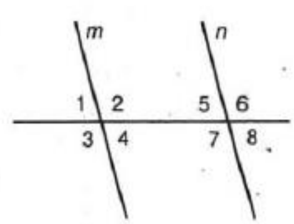
8. a. The measures of the angles of a triangle are $2x - 3$, $3x - 4$, and $4x - 2$. Find the measure of each angle.

____

b. Is the triangle acute, right, or obtuse? ____________

9. If $\angle A$ and $\angle B$ are complementary and the measure of $\angle A$ is four times the measure of $\angle B$, find the measure of $\angle B$. ____________

Exercises 10–12 refer to the diagram at the right.



Exs. 10–12

10. Classify each pair of angles as alternate interior, same-side interior, or corresponding angles.

a. $\angle 1$ and $\angle 5$ ____________ **b.** $\angle 4$ and $\angle 5$ ____________

11. If $m \parallel n$ and $m\angle 1 = 90$, find $m\angle 7$. ____________

12. If $m \parallel n$ and $m\angle 4 = 60$, find $m\angle 7$. ____________

13. Find the measure of each angle of a regular octagon. ____________

REGRESSION!

DIRECTIONS: *This is a reading and exploration exercise. Read through the whole packet with your calculator,* ***placing a large check mark by each statement that asks you to do ANYTHING*** *as you complete each step. Finally,* ***complete the assignment at the end of the reading****.*

Many times in science or in economics, we will have information (data) about a specific phenomenon, but we will not know the underlying relationship that the data represents. A simple model always worth considering is a LINEAR MODEL: $y = mx + b$, where *y* is one variable, *x* is the other, and *a* and *b* are just numbers.

Another common relationship is an EXPONENTIAL MODEL: $y = a \cdot b^x$. In this one, *a* is an initial value, *b* is 1 plus the rate of change (1.05 = 5% growth; .94 = 1 -.06 would be 6% loss or decay), *x* is the time, and *y* would be the final value.

Your graphing calculator is set up to allow you to enter data in lists and determine the equation or model which best represents that data.

Get your graphing calculator out and try to enter this data into the L_1 and L_2 lists:

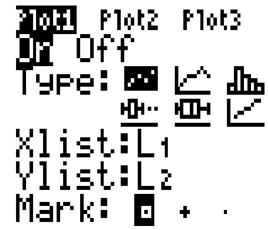
Press STAT then EDIT and put these numbers in:

L_1 (The number of months)	L_2 (Dollars Kate has saved from her allowance)
0	0
1	8
6	43
8	56
13	90

Now, let's get a visualization of this information.

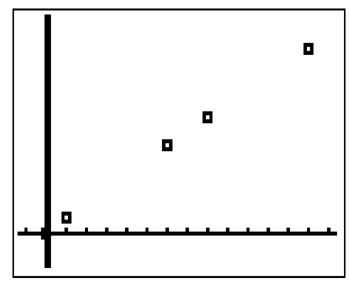
Press these keys: 2^{ND} STATPLOT 1:

Make your screen look like this:



This means "Work with plot number 1, turn plotting on, select a scatterplot, use data from lists L_1 and L_2, and mark data points with a small square." (To enter L_1 or L_2, press 2^{ND} and then the 1 or 2 key.)

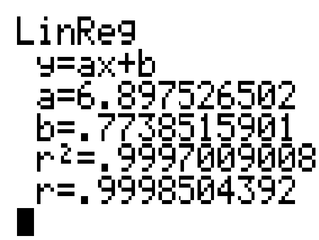
Now, press ZOOM and scroll down to ZOOMSTAT and press ENTER. Your screen should look like this. The points are almost perfectly aligned, so a linear model will describe the relationship between the numbers of months and how much Kate has saved.



Now, we would like to find out what line best "fits" this data. The process of finding a model or equation from data is called **REGRESSION**, and your calculator is programmed to do this!

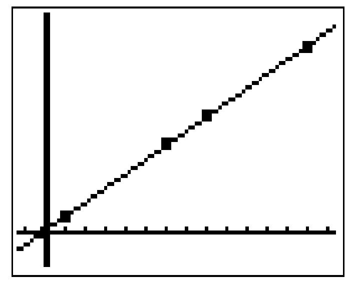
Press STAT, then CALC and select LINREG (ax+b). Now press ENTER.

Your screen should look like this:



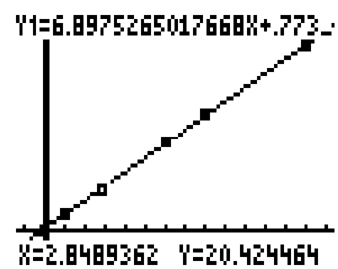
What is the calculator trying to tell us?

- The equation of the "line of best fit" is approximately y = 6.9x + 0.77.
- The "r" value is called the correlation coefficient. The closer it is to 1 (or to -1), the better the fit is. Here, with r at .9998, it is practically a perfect fit. This is very rare!
- If your calculator did not display an "r" value, then follow these steps: Press 2^{ND} CATALOG and scroll down to DIAGNOSTICON. Then, press ENTER.
- Now, rerun the LINREG command, but tack on "L_1, L_2, Y_1" at the end of the command before hitting ENTER. (This will store the regression equation into variable y_1, and when you press ZOOMSTAT, you will see it get graphed along with the data.) To access the y_1 variable, press VARS Y-VARS FUNCTION Y_1. The comma key is one up from 7.



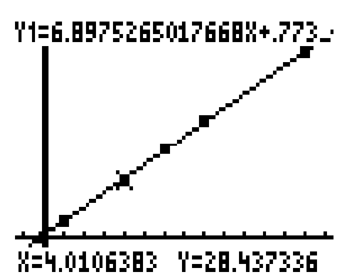
- So, what can we do now? Well, quite a lot, really. Let's start with exploring this graph a bit. Press TRACE and use the right and left arrows to move among the data points. See how the calculator displays the x- and y-coordinates of the points as it jumps among them. Now, try the up and down arrows. These will allow you to move along the line to different points that were not data points.

  Although the blinking cursor doesn't show up on this screen image, it will on the calculator. Whoa, way cool!



Now, suppose you want to determine a guess for how much money Kate had after 4 months. Move the cursor toward an x-value of 4.

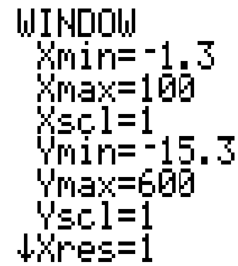
This was the closest I could get. From looking at the x- and y-coordinates displayed below the graph, we can see that that when x is close to 4, y is close to 28. So, our model predicts Kate had about $28 after 4 months.



That was fun, but suppose we want to know when she will have enough to buy an IPAD for $500. But, that y-value is WAY off the scale we have. What to do, eh?

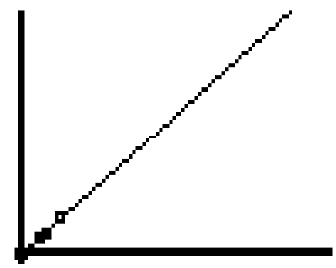
Well, there a few things we could do. Let’s try changing the window to see more of the line!

Press WINDOW…

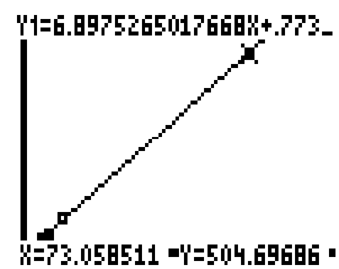


Change your X_{max}-value to 100 and Y_{max} to 600.

Press GRAPH…



Now, try tracing out to a Y-value of 500…

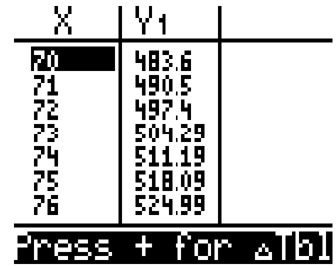


That’s close. So, it looks like she’ll need to save for about 73 months (6 years and a month) to have enough (tax not included)!

How about more accurate predictions? Well, your calculator can make tables with predictive capability. Press 2^{ND} TBLSET and see what you get…

Set TBLSTART to 70 and the delta TBL to 1.

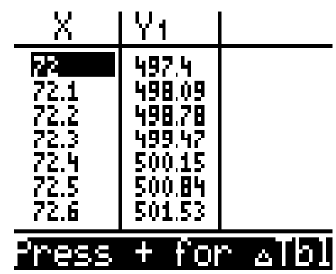
Now, press 2ND TABLE:



Once again, we have confirmation that after 73 months, Kate will be able to buy her iPad.

But, she's anxious to know the **exact** time of that visit. Press 2ND TBLSET again. Change TBLSTART to 72 and delta TBL to 0.1:

Now, Press 2ND TABLE again:



So, 72.4 months will be needed (.4 x 30 = 12 days), so she'll be there in about 6 years and 12 days. I think she's already got an appointment!

Next time out, we'll do some exploring with EXPREG – exponential regression, which will work with curves, not just lines.

ASSIGNMENT: Go online and find a data set with linearly related data values. Put the data into lists in your calculator and play around with the data. Find a linear regression line, describe the meaning of the slope and y-intercept of your regression line given the data you selected, and describe anything interesting you encountered while playing around with the data on your calculator. You can write your work on a separate sheet of graph paper and attach it to this packet. Please include the data set you used and a citation of where you found it. (You may use *Wikipedia* as a starting point for finding more reliable primary sources, but not as your only source for your data.)

Summer Work
Honors Algebra II

Name: ____

Part Two

Honors Algebra Two Students:

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Description of assignment: Each of the four parts of the summer work is split into three sections.

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2. Review of concepts from geometry – **complete all odd problems**
3. Introduction to regression – **complete entirely**

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Algebra Review

Use the distributive property to write an equivalent expression.

1-5 **1.** $-3(6x - 2y + 5z)$ ________ **2.** $8(6c + 7d + 1)$ ________

5-9 **3.** $-4x^3(5x^2 - 7x - 6)$ ________ **4.** $3a^2b(4ab - 3b + 1)$ ________

5-2 Simplify.

5. $(a^4)^3$ ________ **6.** $(-4x^3)^3$ ________ **7.** $(2xy^2)^4$ ________

6-7 Factor.

8. $a^6b^2 - a^5b^3 - a^2b^2$ ________ **9.** $16a^2 - 49y^2$ ________

10. $9x^2 - 60xy + 100y^2$ ________ **11.** $24c^2 - cd - 3d^2$ ________

3-7 Solve the formulas for the given letter.

12. $V = \pi r^2 h$, for h ________ **13.** $E = \frac{1}{2}mv^2$, for m ________

Translate to an equation and find all solutions.

3-1 **14.** Celeste collected 416 aluminum cans. That was 87 more cans than Louis collected. How many cans did Louis collect? ________

6-9 **15.** The square of a number is 3 less than 4 times the number. Find the number. ________

16. The width of a rectangle is 3 cm less than the length. The area of the rectangle is 54 cm^2. Find the width and the length. ________

5-5 Identify the terms. Give the coefficient and degree of each term.

17. $45x^4 - 4y^2$ ________ **18.** $3x^2y^2 - 5x + 9$ ________

7-4 Find the slopes, if they exist, of the lines containing these points.

19. $(5, -1)$ $(5, -6)$ ________ **20.** $(2, 2)$ $(7, 2)$ ________ **21.** $(3, 1)$ $(-1, 2)$ ________

7-5 Find the slope and y-intercept of each line.

22. $3x - 2y = 8$ ________ **23.** $5y - 15 = x$ ________

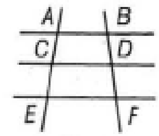
7-8 Determine whether the graphs of the equations are parallel.

24. $y - 2 = -2x$
$3y - 2 = -6x$ ________

25. $2y - 8 = 9x$
$3y - 7 = 4x$ ________

Geometry Review

Directions: Write answers in the spaces provided.

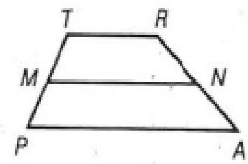


Ex. 1

1. $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$ and $BD = DF$. If $AC = 7$, find AE. ____________
2. Name the property that justifies the statement "If $y + 7 = 19$, then $y = 12$."

   ____

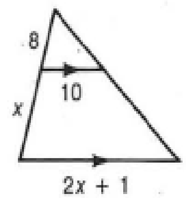
3. $\triangle RGT$ is an isosceles right triangle with right angle G. $\overline{GX}$ is a perpendicular from G to $\overline{RT}$. Which method(s) could you use to prove $\triangle XGR \cong \triangle XGT$? ______________________________



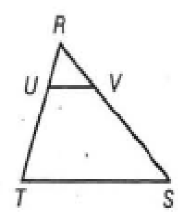
Ex. 4

4. $\overline{MN}$ is the median of trapezoid $TRAP$. If $TR = 16$ and $PA = 38$, find MN. ____________

5. Find the value of x. ____________



6. Assume the two statements "Whenever Jack Laughton drives, we are late" and "We were on time" are true. What, if anything, can you conclude? ______________________________
7. X is a point on $\overline{AE}$ such that $AE = 13$, $AX = 3y - 5$, and $XE = 2y - 2$. Is X the midpoint of $\overline{AE}$? ____________
8. In quadrilateral $ABCD$, $\overline{AD} \parallel \overline{BC}$, $m\angle D = 90$, $m\angle C = x + 15$, and $m\angle B = 2x + 15$. Find $m\angle A$. ____________
9. A, B, and C are in the intersection of planes M and P. What is the relationship between A, B, and C? ____________



Ex. 10

10. In $\triangle RST$, $\overline{UV} \parallel \overline{TS}$. What postulate allows you to conclude that $\triangle RUV \sim \triangle RTS$?

    ____

11. What is the correct first sentence for an indirect proof of the conditional "If $\angle A \not\cong \angle B$, then $\overline{AC} \not\cong \overline{BC}$."

    ____

12. $\angle A$ and $\angle B$ are alternate interior angles formed by a transversal of two parallel lines. If $m\angle A = 12x + 2$ and $m\angle B = 9x + 20$, find the measures of $\angle A$ and $\angle B$. ____________

REGRESSION: EXPONENTIAL GROWTH

DIRECTIONS: *This is a reading and exploration exercise. Read through the whole packet with your calculator,* ***answering all questions thoroughly*** *as you complete each step.*

In the first summer work packet, you learned about linear regression. That means you were using a set of data that was well-suited to be modeled by a line. The truth of the matter is that many quantities in the real world and in nature grow and decay using a different model altogether. For example, consider the population of the world.

Here is some data: *(Source: http://www.census.gov/ipc/www/worldhis.html)*

Historical Estimates of World Population

(Population in millions. When lower and upper estimates are the same they are shown under "Lower.")

Year	Summary		Biraben	Durand		Haub	McEvedy and Jones	Thomlinson		UN, 1973		UN, 1999	USCB
	Lower	Upper		Lower	Upper			Lower	Upper	Lower	Upper		
10000 BC	1	10					4	1	10				
8000 BC	5					5							
6500 BC	5	10								5	10		
5000 BC	5	20					5	5	20				
4000 BC	7						7						
3000 BC	14						14						
2000 BC	27						27						
1000 BC	50						50						
500 BC	100						100						
400 BC	162		162										
200 BC	150	231	231				150						
1 AD	170	400	255	270	330	300	170	200		200	400	300	
200 AD	190	256	256				190						
400 AD	190	206	206				190						
500 AD	190	206	206				190						
600 AD	200	206	206				200						
700 AD	207	210	207				210						
800 AD	220	224	224				220						
900 AD	226	240	226				240						
1000 AD	254	345	254	275	345		265					310	
1100 AD	301	320	301				320						
1200 AD	360	450	400			450	360						
1250 AD	400	416	416									400	
1300 AD	360	432	432				360	400					
1340 AD	443		443										
1400 AD	350	374	374				350						
1500 AD	425	540	460	440	540		425					500	

Year	Summary		Biraben	Durand		Haub	McEvedy and Jones	Thomlinson		UN, 1973		UN, 1999	USCB
	Lower	Upper		Lower	Upper			Lower	Upper	Lower	Upper		
1600 AD	545	579	579				545						
1650 AD	470	545				500	545	500		470	545		
1700 AD	600	679	679				610	600					
1750 AD	629	961	770	735	805	795	720	700		629	961	790	
1800 AD	813	1,125	954				900	900		813	1,125	980	
1850 AD	1,128	1,402	1,241			1,265	1,200	1,200		1,128	1,402	1,260	
1900 AD	1,550	1,762	1,633	1,650	1,710	1,656	1,625	1,600		1,550	1,762	1,650	
1910 AD	1,750											1,750	
1920 AD	1,860											1,860	
1930 AD	2,070											2,070	
1940 AD	2,300											2,300	
1950 AD	2,400	2,557	2,527			2,516	2,500	2,400		2,486		2,520	2,557

Sources:

Biraben, Jean-Noel, 1980, An Essay Concerning Mankind's Evolution,
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Durand, John D., 1974, "Historical Estimates of World Population: An Evaluation,"
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McEvedy, Colin and Richard Jones, 1978, "Atlas of World Population History,"
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Thomlinson, Ralph, 1975, "Demographic Problems, Controversy Over Population Control,"
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U.S. Census Bureau (USCB), 2010, "Total Midyear Population for the World: 1950-2050",
Data updated 12-20-2010,
http://www.census.gov/ipc/www/idb/worldpop.php

Here is some data for the last 60 years or so: *(Source: http://www.worldometers.info/population/)*

Year	World population in millions
1960	3,000
1970	3,700
1980	4,500
1990	5,300
2000	6,100
2005	6,450
2010	6,800

I want to demonstrate to you that a linear model is just not in the realm of the reasonable for this data. So, what I would like you to do is the following:

- Take the data from the first table found by **Haub** (covering years 8,000 BC to 1950 AD) and combine it with the information from the table above (years 1960-2010). Input the data into your calculator, inputting the time (in years) in L_1 and the population (in millions) in L_2. (Remember: you call up the lists by hitting **STAT**, then **EDIT**.)
- Make a scatterplot of years (L_1) and populations (L_2). Use **2ND STATPLOT ON {SCATTERPLOT ICON} L_1 L_2 {SQUARE ICON}**
- Be sure to change your window settings accordingly, maybe -10000 to 3000 for the horizontal (time) axis and from -100 to 7000 for the vertical (population) axis.

Make a sketch of the scatterplot you see:

Do you think that a line would “fit” this data well? Why or why not? For what years might a line be a reasonable model? When does it start to go off track? Comments, thoughts?

It should be clear from the scatter plot that a linear model would not be valid in the long run. The data points do not lie on one line, but we might be able to describe them with a curve. The curve we will use is going to be an **exponential growth curve**. To learn about this sort of curve, let's begin by comparing it to linear models.

In a linear model, the amount *y* goes up or down is directly related to the amount *x* goes up or down. That's what makes the relationship *linear*. Here is the basic equation of a **linear model:**

$$y = mx + b$$

In this model, *y* represents the value of a quantity at time *x*. The value of *b* is the *y*-value when *x* = 0, also known as the "initial" value or the *y*-intercept. The word "initial" here may be a misnomer, because times before 0 could make sense depending on the context of the problem. The *m* value tells us the slope of the line and the rate at which *y* changes with respect to *x*. For example, if *m* = 6, then *y* will increase by 6 units for every one that *x* increases.

In an **exponential model**, things are a bit different:

$$y = a \cdot b^x$$

Here,

a is the initial amount,

b is the growth factor (= 1 + growth rate in decimal or 1 – decay rate in decimal),

and ***y*** is the amount of the quantity at time ***x***.

An example might be:

$$y = 3{,}500(1.048)^x$$

This could be a way to calculate how much an investment of $3500 would be worth, if it grew at a rate of 4.8% per year for *x* years **compounded yearly** (meaning that you start to earn interest on interest only at the end of the year).

Exponential growth occurs anytime the amount something grows is directly related to how much of the stuff there currently is. In the last example, the investment grows at 4.8% annually. That makes it seem like the amount the investment is worth is constantly increasing and that a line would model it well. But, since the total amount is growing all the time, the amount it grows by will keep increasing proportionally to how much there is currently in the account. In year one, you might only earn $168 on the original investment of $3,500. But, in the subsequent years, you will earn more than that because you are earning interest on a higher amount, namely $3,500 plus the interest already earned.

In the world population example, growing by 2% per year means an increase of 20 million people if the current population is 1 billion. But if the current population is 5 billion, the increase would be 100 million!

So, let's create an exponential model for the data.

Press **STAT CALC EXPREG L_1, L_2, Y_1** and hit enter.

Record the model here:

y = ___________________ (__________)^x

Now, press graph and see what you get! If you don't have statplot 1 on, please turn it on (2^{nd} Y=).

Sketch the curve and data points here, please.

Questions:

1. What is the *y*-intercept of the curve?

2. What does that represent?

3. At what yearly rate is the population growing? (Remember to subtract 1 from the factor in the parentheses.)

4. If the *b*-value of your model ($y = a \cdot b^x$) were 2.13, what would that mean?

5. According to the model, not the data, when does the population get up to 3 billion? Did you use TRACE or a TABLE?

6. Why might the data and the model differ?

7. Predicting beyond either end of the data is called EXTRAPOLATION. Predictions made within the domain of the data are called INTERPLOATIONS. Make a prediction about the world population in the years 1160, 2020 and 2050. Describe how you did this. (Consider using TRACE or a calculator TABLE.)

Summer Work

Honors Algebra II

Name: ____

Part Three

Honors Algebra Two Students:

The following questions are designed to keep your math skills sharp over the lazy summer months. The summer work packet is divided into four parts, and each part will require about 2 hrs to complete. In addition to providing you with practice while away from school, this review will help me assess your strengths and weaknesses coming into the class. While you are free to use any notes or texts from previous classes, please do not ask anyone else for assistance. Further expectations for the work's completion are outlined below. Take note of those problems with which you have trouble; we will have the opportunity to discuss tricky problems during the first week of class.

Description of assignment: Each of the four parts of the summer work is split into three sections.

1. Review of concepts from algebra – **complete all odd problems**
2. Review of concepts from geometry – **complete all odd problems**
3. Introduction to regression – **complete entirely**

If you really get stuck on one of the assigned problems, do another problem in its place and write a note to me indicating your substitution. You are encouraged to do more than the assigned problems, if your heart desires—the more you practice, the sharper you'll be.

Notes regarding completion and submission:

1. All parts of the summer work must be submitted on the first day of class.
2. You may use a calculator.
3. All work must be done in **pencil**. If you need additional pages to do your work, **you must use graph paper**—sorry, math department policy—and be sure to include these extra pages when submitting your summer work.
4. Please **box** or **circle your answers**.
5. **Show your work!** I am interested in both **accuracy** and **process**.
6. You may use any written resources available to you, but please do not ask another person for assistance.
7. **Grades are based on effort, not correctness**, so give everything your best shot!
8. The grade you earn will play a significant role in your first marking period grade and effort grade.

Algebra Review

7-8 Determine whether the graphs of the equations are parallel, perpendicular, or neither.

1. $y = 3 + 5x$
$3x - y = -2$ ________

2. $-3x + 6y = 2$
$y = -2x - 10$ ________

3. $x + 6 = y$
$y - x = -2$ ________

5-3 Simplify.

4. $\dfrac{-12a^2bc^4}{-3abc^2}$ ______ **5.** $\dfrac{m^6}{m^4}$ ________ **6.** $\dfrac{9x^3y}{3x}$ ________ **7.** $\dfrac{t^3}{t^3}$ ________

5-4 Write using scientific notation.

8. 0.00381 ________ **9.** 17.662 ________ **10.** 36,840,000 ________

6-7 Factor.

11. $x^2 - 15x + 54$ ________

12. $8a^2 + 22a + 15$ ________

13. $m^3 - 2m^2 + 3m - 6$ ________

14. $18t^2 - 128$ ________

15. $3y^2 - 20y + 12$ ________

16. $x^2y^2 + 8xy + 12$ ________

3-7 Solve for x.

17. $abx = c$ ________

18. $x - m = 15$ ________

19. $3a - 4x = 8$ ________

20. $4x + 5 = cx + 2$ ________

21. $ax - 3x = 5$ ________

22. $mx^2 = 5x$ ________

Solve.

8-2 **23.** $5m + n = 8$
$3m - 4n = 14$ ________

24. $3b - a = -7$
$5a + 6b = 14$ ________

8-3 **25.** $x + y = 9$
$2x - y = -3$ ________

26. $2x + y = 6$
$x - y = 3$ ________

27. The difference between two numbers is 11. Twice the smaller plus three times the larger number is 123. What are the numbers? ________

8-5 **28.** A car leaves Sacramento for Los Angeles traveling 54 mi/hr. A half hour later another car leaves Sacramento for Los Angeles traveling 64 mi/hr. How long after the second car leaves will it pass the first car? ________

9-2 **29.** $5 < 2x + 9 \leq 15$ ________

30. $-7 \leq 3x - 1 < 5$ ________

Geometry Review

Directions: Write answers in the spaces provided.

1. Find the geometric mean between 5 and 12. ________________
2. A and B are points in a plane. If the locus of points in that plane that are 4 cm from A and 3 cm from B contains exactly 2 points, what can you conclude about the length of $\overline{AB}$? ________________
3. Name the property that justifies the statement, "If $3z + x = 12$ and $x = 6z$, then $3z + 6z = 12$." ________________
4. In $\triangle RST$, $m\angle R = 5y - 9$, $m\angle S = 8y + 1$, and $m\angle T = 12y - 12$. Name the shortest side of $\triangle RST$. ________
5. Find the length of a side of a square with diagonal of length 5. Express your answer in simplest radical form. ________________
6. $\triangle LMN \sim \triangle PQR$, $LM = 81$, and $PQ = 18$. Find the ratio of the perimeter of $\triangle LMN$ to the perimeter of $\triangle PQR$. ________________
7. ABC is a triangle with medians $\overline{AM}$, $\overline{BN}$, and $\overline{CP}$ intersecting at O. If $CO = 4x$ and $OP = x + 1$, find the value of x. ________________
8. $\overleftrightarrow{TU}$ is tangent to $\odot O$ at U. A secant from T to $\odot O$ intersects the circle at V and W. If $TV = 4$ and $VW = 12$, find TU. ________________
9. X, Y, and Z are points on a number line with coordinates -6, 4, and 14, respectively. Find the distance from X to the midpoint of $\overline{YZ}$.

   ____
10. Write the inverse of the statement: "If $x = -2$, then $x^2 = 4$."

    ____
11. Find the value of x if $\frac{x+1}{x+3} = \frac{x+4}{x+8}$. ________________
12. A rectangle has length 12 and width 5. Find the length of a diagonal of the rectangle. ________________
13. $\overline{AB}$ is a chord of $\odot O$. The distance from O to $\overline{AB}$ is 5. If the radius of $\odot O$ is 9, find AB in simplest radical form.

    ____
14. Find the measure of an exterior angle of a regular polygon with 24 sides. ________________
15. In $\triangle YES$, $\overline{YE} \cong \overline{YS}$ and $m\angle S = 38$. If $m\angle Y = 6t + 14$, find the measures of $\angle Y$ and $\angle E$. $m\angle Y$________, $m\angle E$ ________

REGRESSION: EXPONENTIAL DECAY

DIRECTIONS: *This is a reading and exploration exercise. Read through the whole packet with your calculator,* ***answering all questions thoroughly*** *as you complete each step.*

Here is some data about the value of a boat versus how old the boat is:

Age	Value in dollars
New	168,000
1 year	163,000
3 yrs.	155,000
6 yrs.	143,000
11 yrs,	124,000
19 yrs.	100,000
27 yrs.	80,000
37 yrs.	61,000

Input the data into L_1 and L_2. What do you see when you graph the data? Quick sketch and description…

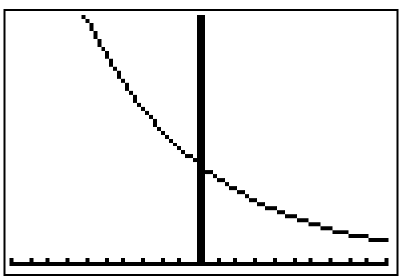
Think about the actual situation we are describing… the value of an aging boat. Do you think the value will ever be zero? Will the value approach zero? Thoughts?

Does a linear model for this situation make sense? Why or why not?

It is time for us to consider an alternative to a steadily declining (which is to say negatively sloped) line. If we use the exponential model:

$$y = a \cdot b^x$$

and we let *a* be a positive quantity, like 400, and we let the *b* value be something less than 1, like .86, then the curve that is generated will look something like this one:



$y = 400(0.86)^x$

Now, this looks a little familiar, no?

Compare the curve to the data points you have… Looks like we could have a decent fit if we could find the values for *a* and *b*. Fortunately, that's what regression does!

So, please press STAT CALC EXPREG L_1, L_2, Y_1 to perform exponential regression and store the generated equation directly into the variable y_1. Record the model here:

y = ____________________ (__________)^x

What do you think the *a* and *b* values are trying to tell us?

Did you guess that *a* is the initial value (selling price) of the boat?

The meaning of *b* is related to the fact that the boat loses value over time. The quantity 1 – *b* is the rate at which the boat loses value per year. So, a value of 0.785 would mean that the boat is losing 21.5% of its value every year (1 - 0.785 = 0.215 = 21.5%).

What was the correlation coefficient on this regression? (That's the *r* value, in case you forgot.) What does that suggest about the model and the data?

Using the TABLE capabilities of the calculator, predict the value of the boat at 10 year anniversaries, so after 10, 20, 30, … years. Stop at age 100 years. Record your predictions here:

10 yrs	20	30	40	50	60	70	80	90	100

On a separate sheet of graph paper, please plot your predictions and try to draw a smooth curve through them. Add a point to represent the initial value, too.

Let's think about the actual situation… an aging boat. What do you know about antique cars? Are they worth a lot if they are well cared-for? Do you think this also applies to boats?

Given what you found out, do you think an exponential model is a good one for the value of a boat? Would your answer be any different if you knew that most boat owners junk their boats in less than 30 years?

I hope that whetted your appetite for another application!

Most of the 92 naturally occurring elements (like oxygen, hydrogen, carbon, gold, helium, silver, etc.) come in several varieties called isotopes. Isotopes of a given element have the same number of protons in their nuclei, but differ in the number of neutrons the nuclei have. For example, although every atom of carbon in the universe has 6 protons in its nucleus, some have 6 neutrons, some have 7, and some have 8. In fact, there is an isotope of carbon with as few as 2 neutrons in it and one with as many as 16. The total number of protons plus neutrons gives the isotope its number. So, carbon-12 has six of each, whereas carbon-13 has 6 protons and 7 neutrons.

The 6 neutron variety is by far the most common, almost 99% of all carbon is this isotope. Carbon-12 and Carbon-13 are very "stable", meaning that these isotopes almost never change into another element. The process of changing from one element into another is called ***radioactive decay***. Carbon-14 undergoes such a transformation. The rate at which an isotope changes into other elements is usually described with one number: the "half-life" of the isotope. The half-life is the time it takes for exactly half of the isotope to change into other (usually lighter) elements. Half lives can be very short (nanoseconds or less) or long (thousands or millions or even billions of years).

Carbon-14's half-life is 5730 years, making it a superb choice for dating artifacts up to about 60,000 years old. By comparing how much carbon-14 is in a similar object made today to how much is in the artifact, scientists can determine how many half-lives have passed since the artifact was created. This is called ***radiocarbon dating*** or just ***carbon dating.***

Here is some data on a sample of new element called HAMILTONIUM-63. It decays fairly rapidly…

Date of measurement	Amount of sample (in grams)
Sept.8	355
Sept.11	334
Sept. 15	308
Sept. 18	290
Sept. 23	262

Use this data to create an exponential decay model. Record the model here:

Use a calculator table to estimate the half-life of Hamiltonium-63.

Summer Work

Honors Algebra II

Name: ____

Part 4

Honors Algebra Two Students:

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Algebra Review (Page 1)

6-2 Which of the following are differences of two squares?

1. $10x^2 - 100$ ______ **2.** $9x^2 + 81$ ______ **3.** $16x^2 - 25$ ______

4. $a^2b^2 - a^2c^2$ ______ **5.** $49 - x^2$ ______ **6.** $121y^2 - 80$ ______

7-6 Write an equation for each line that contains the given pair of points.

7. $(-3, -2)$ $(2, 13)$ ______ **8.** $(-8, -3)$ $(4, 2)$ ______

10-5 Find the least common multiple (LCM).

9. $5m + 15, m^2 - 9$ ______ **10.** $y - 3, 9 - 3y$ ______ **11.** $a^2 - 25, a + 5$ ______

11-5 Rationalize the denominator.

12. $\frac{\sqrt{6}}{\sqrt{5}}$ ______ **13.** $\frac{\sqrt{10}}{\sqrt{3}}$ ______ **14.** $\frac{\sqrt{27}}{\sqrt{3}}$ ______ **15.** $\frac{\sqrt{2}}{\sqrt{7}}$ ______

16. $\sqrt{\frac{x}{3}}$ ______ **17.** $\sqrt{\frac{5}{y}}$ ______ **18.** $\frac{\sqrt{24c^3}}{\sqrt{6}}$ ______ **19.** $\frac{\sqrt{18y}}{\sqrt{2}}$ ______

Factor.

6-7 **20.** $7a^2 - 14a + 49$ ______ **21.** $m^2 - 5m - 36$ ______

22. $-3x^2 - 3x + 18$ ______ **23.** $x^3 + 3x^2 - 10x$ ______

11-3 **24.** $-\sqrt{72}$ ______ **25.** $\sqrt{25y^2}$ ______

26. $\sqrt{81m}$ ______ **27.** $\sqrt{x^2 - 10x + 25}$ ______

5-4 Write using scientific notation.

28. 17,430 ______ **29.** 0.0301 ______ **30.** 0.000009726 ______

Multiply and collect like terms.

5-9 **31.** $(3x - 7)(5x + 2)$ ______ **32.** $(3m^2 + 2)(m^2 + 6)$ ______

5-11 **33.** $(x^2 - 6x + 9)(x - 2)$ ______ **34.** $(5y^2 - 4)(2y^2 + 11y - 1)$ ______

11-4 **35.** $\sqrt{xy}\sqrt{yz}$ ______ **36.** $\sqrt{2a}\sqrt{8a}$ ______ **37.** $\sqrt{3y^3}\sqrt{8y^4}$ ______

12-1 Find the indicated outputs for these functions.

38. $f(t) = t^3 + t + 1$; find $f(0), f(-2), f(2)$ ______

39. $f(b) = b^2 + b - 2$; find $f(-1), f(1), f(-4)$ ______

Algebra Review (Page 2)

9-1 Write using roster notation.

1. The set A of all integers that are perfect squares between 20 and 100 ________________

2. The set B of all positive integer factors of 36 ________________

3. The set C of all integers that are multiples of 5 between -18 and 23 ________________

Divide.

10-3 **4.** $\frac{5y-5}{2} \div \frac{y-1}{8y}$ ________________ **5.** $\frac{mn+n^2}{m} \div \frac{m^2-n^2}{mn^2}$ ________________

10-9 **6.** $(x^2 - 7x + 3) \div (x - 2)$ ________________ **7.** $4y^2 + 18y - 9 \div (2y + 1)$ ________________

11-5 **8.** $\frac{\sqrt{75}}{\sqrt{3}}$ __________ **9.** $\frac{\sqrt{36y}}{\sqrt{9}}$ __________ **10.** $\frac{\sqrt{99c}}{\sqrt{11}}$ __________ **11.** $\frac{\sqrt{56y^3}}{\sqrt{7}}$ __________

11-7 Use the Pythagorean theorem to find the hypotenuse (c) or the legs (a and b) of a right triangle.

12. $c = 15, a = 9, b =$ ________________ **13.** $a = 5, b = 12, c =$ ________________

14. $a = \sqrt{5}, c = \sqrt{11}, b =$ ________________ **15.** $b = 7, c = 7\sqrt{2}, a =$ ________________

5-4 Write using standard notation.

16. 1.6038×10^{-4} ________________ **17.** 7.6623×10^{6} ________________

Solve.

8-3 **18.** The sum of two numbers is 32. One half the first number plus one third of the second number is 14. Find the numbers. ________________

10-8 **19.** Nut mix A is 40% peanuts and nut mix B is 65% peanuts. How much of each is needed to make 40 lbs of a mix that is 55% peanuts? ________________

11-2 Determine the replacements for x that make the expression a real number.

20. $\sqrt{x-7}$ __________ **21.** $\sqrt{x^2+1}$ __________ **22.** $\sqrt{2x}$ __________ **23.** $\sqrt{x^2-2}$ __________

12-3 Write a linear function and solve.

24. Jerome earned $4.00 an hour for cleaning an attic, plus a bonus of $5.00. He worked for 5 hours. How much did he earn?

____

25. Twyla bought 5 yards of ribbon for $2.50 a yard, plus there was a flat service charge of $2.00. What was the total cost of the ribbon?

____

Geometry Review

16. In $\triangle XYZ$, A and B are the midpoints of $\overline{XY}$ and $\overline{YZ}$, respectively.

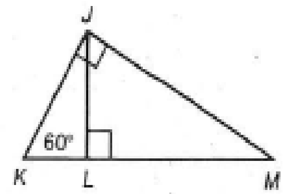
    If $AB = 5n - 12$ and $XZ = 4n - 6$, find the value of n. ________

17. Find the length of an altitude of an equilateral triangle with sides

    of length 8. Express your answer in simplest radical form. _______

18. In the diagram, $JK = 1$. Find KL, LM, JL, and JM.

    $KL =$ ________, $LM =$ ________, $JL =$ ________, $JM =$ ________



Ex. 18

19. P, Q, R, and S are points on $\odot O$ such that the secants $\overleftrightarrow{PQ}$ and $\overleftrightarrow{SR}$ intersect outside the circle. Find the measure of the acute angle

    formed if $m\widehat{PS} = 110$ and $m\widehat{QR} = 50$. ____________________

20. In quad. $ABCD$, $\overline{AB} \parallel \overline{DC}$, $m\angle A = 14x + 3$, $m\angle D = 3x + 7$, and

    $m\angle C = 8x - 5$. Find $m\angle B$. ____________

21. $JKLM$ is a parallelogram. If $m\angle J = 24$, find $m\angle L$ and $m\angle M$.

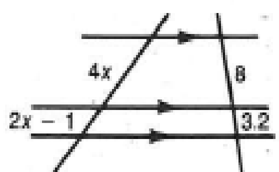
    $m\angle L =$ ____________, $m\angle M =$ ____________

22. The ratio of the measures of two supplementary angles is

    4:5. Find the measure of each angle. ____________

23. Find the value of x.

    ____



Ex. 23

24. The bases of an isosceles trapezoid have lengths 4 and 10. If the

    median of the trapezoid has length $2x + 1$, find the value of x. ____________

25. $\overline{XY}$ is a diameter of $\odot O$ and Z is a point on $\odot O$ such that

    $m\angle XYZ = 42$. Find $m\widehat{ZY}$. ____________

In Exercises 26–28, use a scientific calculator or the table on page 311 of the textbook.

26. A building casts a 20 ft shadow when the angle of elevation of the sun is 42°. Find the height of the building to the nearest foot.

    ____

27. In $\triangle RST$, $m\angle S = 90$, $RS = 44$, and $TS = 52$. Find $\angle T$ to the

    nearest degree. ____________

28. In $\triangle XYZ$, $m\angle Z = 90$, $m\angle X = 62$, and $XY = 50$. Find XZ and

    ZY to the nearest tenth. $XZ =$ ____________, $ZY =$ ____________

29. Name the largest angle of $\triangle PQR$ if $PQ = 3x - 2$, $QR = 2x - 2$,

    $PR = 2x + 6$, and the perimeter of $\triangle PQR$ is 86. ____________

REGRESSION: QUADRATIC RELATIONS

<u>DIRECTIONS:</u> *This is a reading and exploration exercise. Read through the whole packet with your calculator, **answering all questions thoroughly** as you complete each step.*

OK, time for some fun! Go outside, turn the hose on, and take a picture of the stream of water that sprays out and lands on the ground. (If you can't do this for any reason, go online and find a picture of water squirting out of a hose.) Print the picture and attach it to this packet.

The shape the stream of water takes is called a parabola. The equation which models the behavior of any object which is only accelerating due to gravity (we call that **free fall**) is the following:

$$h = \tfrac{1}{2} g t^2 + v_0 t + h_0$$

or height at time t = ½ (acceleration due to gravity) time^2 + (initial velocity)(time) + initial height.

(This equation can also be written with *y* and *x* as follows: $y = \tfrac{1}{2} g x^2 + v_0 x + y_0$. This equation is a perfect example of the intersection of math and physics, and if you think hard you'll realize that you've seen this before—it's what Mr. Gallagher calls "one-stop shopping.")

By analyzing the position versus the time of an object in "free fall", we can learn about its motion. Here is some data about a ball thrown straight up:

time (seconds)	height (meters)
0	7
1	35
2	53
3	62
4	60
5	50
6	29

Please use your calculator to enter and graph this data. Make a sketch here:

So, let's now try to have the calculator create a model which will describe the dataset and allow us to make predictions. We are going to need a new kind of regression – QUADRATIC REGRESSION. The procedure will be the same as before with only one change:

Press STAT CALC QUADREG L_1, L_2, Y_1.

Record your model here:

Now, press GRAPH… sketch please! Does it dip into quadrant 4 at all?

Ok, it's time to analyze the results.

Match up the terms from the regression with the equation $h = \frac{1}{2}gt^2 + v_0 x + h_0$ *(y = h, x = t, etc.)*

What matches up with ½ g?

So, what is the value of g? Is it positive or negative?

What is the value of v_0? (+ or -?)

How about h_0?

Prediction time!

When does the ball hit the ground? How did you get your answer?

When does it reach the highest point? Explain.

At what times does the ball have an approximate height of 45 meters?

Finally, what is the domain of valid times for this model? In other words, what values of t make sense? Please explain your answer.